

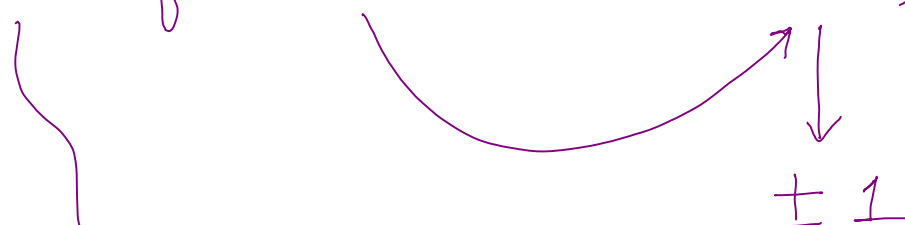
BER of CDMA
System.

consider a single user
CDMA system.

↓
user 0

Transmitted symbol = a_0 .

Code of user 0 $c_0(n)$



± 1

code length = N

Transmitted symbol $x(n)$

$$x(n) = a_0 c_0(n)$$

symbol $\times$ code.

Received symbol $y(n)$

$$y(n) = \underset{\substack{\nearrow \\ \text{Fading Coefficient}}}{h} x(n) + w(n)$$

Each $w(n)$ is
IID Gaussian noise
with variance σ^2

$$E\{w(n)\} = 0$$

Independence

$$\downarrow E\{w(n) \cdot w^*(m)\} = 0 \\ \text{if } n \neq m$$

$$y(n) = h x(n) + w(n)$$

$$y(n) = h a_0 c_0(n) + w(n)$$

↑
system model for CDMA

At the receiver, correlate
with code of user 0.

$$\underbrace{\frac{1}{N} \sum_n y(n) c_0(n)}_{\text{correlation with code } c_0(n)} = \frac{1}{N} \sum_n \left(h a_0 c_0(n) + w(n) \right) \times c_0(n)$$

$$= \frac{1}{N} \sum_n h a_0 c_0^2(n) + \frac{1}{N} \sum_n w(n) c_0(n)$$

$$= h a_0 \underbrace{\frac{1}{N} \sum_n c_0^2(n)}_{\text{Signal part}} + \underbrace{\frac{1}{N} \sum_n w(n) c_0(n)}_{\text{noise part}}$$

$$\frac{1}{N} \sum_n c_0^2(n)$$

Each $c_0(n) = \pm 1$
 $\Rightarrow c_0^2(n) = 1$

$$= \frac{1}{N} \sum_n 1 = \frac{1}{N} \cdot N = 1$$

$$y(n) = \underbrace{h a_0}_{\text{Signal}} + \underbrace{\frac{1}{N} \sum_{n=0}^{N-1} w(n) c_0(n)}_{\text{Noise}}$$

$$\tilde{W} = \frac{1}{N} \sum_n W(n) C_0(n)$$

$$\begin{aligned} E\{\tilde{W}\} &= E\left\{\frac{1}{N} \sum_n W(n) C_0(n)\right\} \\ &= \frac{1}{N} \sum_n \underbrace{E\{W(n)\}}_0 C_0(n) \end{aligned}$$

$$E\{\tilde{W}\} = \frac{1}{N} \times 0 = 0$$

$$\begin{aligned}
E\{\tilde{w}\tilde{w}^*\} &= E\left\{\frac{1}{N}\sum_n w(n) c_0(n) \times \frac{1}{N}\sum_m w^*(m) c_0^*(m)\right\} \\
&= E\left\{\frac{1}{N^2}\sum_n\sum_m w(n)w^*(m) c_0(n) c_0^*(m)\right\} \\
&= \frac{1}{N^2}\sum_n\sum_m \underbrace{E\{w(n)w^*(m)\}}_{\sigma^2} c_0(n) c_0^*(m) \\
&= \frac{1}{N^2}\sum_n \underbrace{E\{|w(n)|^2\}}_{\sigma^2} \underbrace{c_0^2(n)}_1
\end{aligned}$$

$$= \frac{1}{N^2} \sum_n \sigma^2 \cdot 1$$

$$= \frac{1}{N^2} \cdot \sigma^2 N = \frac{\sigma^2}{N}$$

$$E\{|\tilde{W}|^2\} = \frac{\sigma^2}{N} \leftarrow \begin{array}{l} \text{noise power} \\ \text{is decreased} \\ \text{by a factor} \\ \text{of } N. \end{array}$$

$$y(n) = h a_0 + \underbrace{\frac{1}{N} \sum_n w(n) c(n)}_{\tilde{W}}$$

$$= \underbrace{h a_0} + \tilde{W} \quad \text{variance} = \sigma^2/N$$

$$\begin{aligned}
 \text{Signal Power} &= E\{|h|^2 |a_0|^2\} \\
 &= |h|^2 E\{|a_0|^2\} \\
 &= |h|^2 P
 \end{aligned}$$

$$\text{SNR} = \frac{|h|^2 P}{\sigma^2/N} = \underbrace{N}_{\text{Spreading Gain}} \cdot \frac{|h|^2 P}{\sigma^2}$$

Processing Gain

$$\text{SNR}_r = |h|^2 \cdot \frac{NP}{\sigma^2}$$

$$= |h|^2 \cdot N \cdot \frac{P}{\sigma^2}$$

SNR

Exact $\rightarrow$ $= |h|^2 \cdot N \cdot \text{SNR}$

approximate BER $\rightarrow$ $\approx$

$$\text{BER} = \frac{1}{2} \left(1 - \sqrt{\frac{N \cdot \text{SNR}}{2 + N \cdot \text{SNR}}} \right)$$

$$\approx \frac{1}{2 \cdot N \cdot \text{SNR}} \quad \text{at high SNR}$$

Example: For BER of CDMA System.

Consider CDMA system with

$$N = 256. \quad \text{SNR}_{\text{dB}} = \frac{P}{\sigma^2} = 15 \text{ dB}$$

What is BER of CDMA system over a Rayleigh Fading channel?

$$\text{BER} = \frac{1}{2} \left(1 - \sqrt{\frac{N \cdot \text{SNR}}{2 + N \cdot \text{SNR}}} \right)$$

$$N = 256$$

$$\text{SNR}_{\text{dB}} = 15$$

$$10 \log_{10} \text{SNR} = 15$$

$$\text{SNR} = 10^{1.5} = 31.623$$

$$BER = \frac{1}{2} \left(1 - \sqrt{\frac{256 \times 31.623}{2 + 256 \times 31.623}} \right)$$

$$BER = 6.175 \times 10^{-5}$$